Math in the Real World

Lesson 1 -- Models of Real-World Situations Using Families of Functions

Objectives:

Students will be able to:

- Choose an appropriate function family to model a real world situation for linear and quadratic functions
- Change a generic formula to model a situation
- Make inferences about the situation from the information given in the formula and graph

Standards:

A2.4 Models of Real-World Situations Using Families of Functions

A2.4.1 Identify the family of function best suited for modeling a given real-world situation

A2.4.2 Adapt the general symbolic form of a function to one that fits the specifications of a given situation by using the information to replace arbitrary constants with numbers

A2.4.3 Using the adapted general symbolic form, draw reasonable conclusions about the situations being modeled

A3.1 Lines and Linear Functions

A3.1.1 Write the symbolic forms of linear functions (standard, point-slope and slope-intercept) given appropriate information and convert between froms

A3.1.2 Graph lines (including those of the form $x = h$ and $y = k$) given appropriate information

A3.1.3 Relate the coefficients in a linear function to the slope and x- and y- intercepts of its graph

A3.3 Quadratic functions

A3.3.1 Write the symbolic form and sketch the graph of the quadratic function given appropriate information

A3.3.2 Identify the elements of a parabola (vertex, axis of symmetry, direction of opening) given its symbolic form or its graph, and relate these elements to the coefficient(s) of the symbolic form of function.

A3.3.5 Express quadratic functions in vertex form to identify their maxima or minima and in factored form to identify their zeros

What students already know:

Students will have previously learned about all of the function families, their generic graphs and transformations of those graphs. We will have practiced in class modeling a real world situation with mathematics

What students are learning / practicing:

Students will practice modeling real world situations with mathematics. They will analyze how their models differ from the simplest form of the function family they are using. Students will also have to come up with their own context that could be described using the same function family.

What students still need to learn:

This would be an assignment near the very end of an algebra I class. Students would most likely make connections forward with regression for function fitting. This is a good foundation for deeper exploration of families of functions and graphing in algebra II.

Materials:

- Student Page attached

Preparation:

- The teacher should complete the student page
- The teacher should have several examples ready of each family function for struggling students

Ways to assist students who are struggling:

- Encourage the students to think of the problems as real world problems and not math problems. In a conversational tone talk about how you would approach the problems in real life. Ex) "If you were baking cookies and you wanted to double the recipe, what would you do to each of the ingredients?"
- Encourage pictures
- If they are struggling with the math portion, try to relate to a similar problem seen previously
- If they are struggling with the context, try to explain the context in a new way

Ways to assist students who are excelling:

- The open ended final question will challenge students who are excelling to go above and beyond the examples I've provided. It is my general assumption that those excelling will attempt the extra problem if time allows and those who are struggling won't.
- Also ask students who are excelling if they can make the question even more general and/or complicated. For example, how do I change *any* ingredient or what if my painting was *y* feet from the ceiling?
- Ask the students to come up with other real world examples that can be modeled with families of functions they know.

Comment [C1]:
To include on my completed page as teacher resource:
more contextual connections? bags of chocolate chips, calories, # of cookies from a bag vs how many c of choc chips in the recipe, # of choc chips…

OR: break into 2 questions. One looking for other possible relationships, the next pick one to work out a rule/expression

p.s. I intend on completing it by hand and scanning it in once I've settled on a "final" lesson.

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<u>OPTION</u>: For **each** question, you may choose to answer parts A-G **or** you may choose to answer B twice and omit C-G. Be sure to ask and answer quality questions about your relationship in part B. See my questions if you need examples, but do not feel restricted to asking only the questions I've asked. Use the additional blank pages if you need more space.

1. My favorite chocolate chip cookie recipe makes 3 dozen cookies. The recipe is as follows:

¾ cup vegetable shortening	1 ¾ cup all purpose flour
1 ¼ cups firmly packed light brown sugar	1 tsp. salt
2 tbsp. milk	¾ tsp. baking soda
1 tbsp. vanilla extract	1 cup semi-sweet chocolate chips
1 large egg	



1 tablespoon of chocolate chips

A. What questions come to your mind from the given information? What are some relationships you can explore?

Comment [C2]: Include on my completed page: It may be a good idea to allow some time in class to brainstorm relationships in groups

B. Choose one **relationship** from A to explore in detail.
   - Ask and answer several questions about this relationship.
   - Create a model for your relationship.
   - What function family does this model belong to?
   - How does your model differ from the standard form of this function family?

C. How many cups of chocolate chips would I need for 6 dozen cookies? 2 dozen cookies? 6 cookies?

http://whatscookintew.blogspot.com/2011/09/chocolate-chip-cookies.html

D. I like my cookies to be extra chocolaty. Therefore I always add ½ cup more chocolate chips than the recipe asks for. How many cups of chocolate chips would I add for 3 dozen cookies? 6 dozen? 6 cookies?

E. Write a model for calculating how many chocolate chips I would add for *x* cookies.

F. What function family does this model belong to?

G. How does your expression differ from the simplest form of the equation? How do the graphs differ?

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2. I purchased a painting in Africa (for some old clothes, a few odd things in my backpack and 10,000 tsh, which is about $7 – best deal by far!) that is very large. About 3 ft x 5 ft. My uncle recently framed it for me and I just hung it in my apartment!

A. What questions come to your mind from the given information? What are some relationships you can explore?

Comment [C3]: Include on my completed page: It may be a good idea to allow some time in class to brainstorm relationships in groups

B. Choose one **relationship** from A to explore in detail.
   a. Ask and answer several questions about this relationship.
   b. Create a model for your relationship.
   c. What function family does this model belong to?
   d. How does your model differ from the standard form of this function family?

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The room looks a little bare around the painting. I want to make a collage of pictures and smaller art pieces I purchased in Tanzania.

C. The room is 14 feet wide. If I want to have pictures around the edge so they two feet from each wall, the total collage is half as tall as it is wide, how much area do I have to create my collage in?

D. Write a model for how much area I have to work with for a room that is *x* feet wide. (Assume I still want 2 feet of space between the collage and the walls and the collage still needs to be twice as wide as it is tall).

E. What function family does this model belong to?

F. How does your expression differ from the simplest form of the equation? How do the graphs differ?

G. What is another real world example that fits this function family? If time allows, explore modeling this example.

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SAMPLE answers

1.
    A.
    B.
    C. 6 dozen → 2 cups
       2 dozen → 2/3 cups
       6 cookies → 1/6 cups
    D. 3 dozen → 1 ½ cups
       6 dozen → 2 ½ cups
       2 dozen → 7/6 cups (1.17)
       6 cookies → 2/3 cups
    E. Chocolate chips = 1 cup / 36 cookies * x cookies + ½ cup
       $CC = \frac{1}{36}x + \frac{1}{2}$
    F. Linear
    G. A vertical shrink of factor 1/36th (or a horizontal stretch of factor 36)
       A vertical shift of ½ units upwards
2.
    A.
    B.
    C. $(14 - 2 * 2)\left(\frac{14-2*2}{2}\right) - \ 3 * 5$
       $10 * 5 - 15 = 50 - 15 = 35$
       35 ft^2
    D. Area = $(x - 4)\left(\frac{x-4}{2}\right) - 3 * 5$
       Area = $\frac{1}{2}(x - 4)^2 - 15$
       Multiplied out: Area = $\frac{1}{2}x^2 - 4x - 7$
    E. Quadratic
    F. Horizontal stretch of factor 2 (or vertical shrink of factor ½)
       Horizontal shift of 4 units right
       Horizontal shift of 15 units down

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Reflection

I knew this was going to be hard but it's turning out to be much harder than I had originally anticipated. First I had this wonderful idea for a lesson and got almost an entire student page finished only to look back at my standards and realize that the lesson didn't fit the objectives at all. My first go around I was asking students to come up with examples. Although I think that's a very valuable skill it didn't align with my standards so I took a step back and started coming up with some examples on my own. No worries, I saved the "mistake" lesson because it should come in handy later. If not for this project, I can definitely in my classroom.

After that little mishap I worked on actually coming up with real world situations that could be modeled by different families of functions. It's actually quite difficult. I worked with square root functions in my teaching high school math class a few semesters back so I decided to start there. During that class I wrote a unit plan on the square root function (not something I'm that proud of today, but we all have to start somewhere). I remember writing a problem about a referee on a soccer field. I decided to adapt that problem for this project. I need a little help seeing as it is only my first problem in my first lesson.

Now that I was on track with my topic, I needed to try to think of questions that would challenge and engage my students while allowing them to grow as mathematicians. I could think of several questions I could ask about the diagonal of the soccer field, but what questions are engaging? What questions are relevant? And what questions are actually going to help my students learn? Writing the questions is a lot harder than I originally thought. Sure I can mimic the type of questions I've been asked my whole life, and I love math, so they're good, engaging questions, right? Wrong. I know that the questions I think are fun aren't necessarily fun to my friends. To illustrate my point – I got a little tipsy and started doing optimization problems a couple weeks ago. To me, yeah that was a great time. That doesn't mean I'm going to ask my students to do those problems. I was working on 1-30 odds. So, what do my students find interesting? This is quite difficult with hypothetical students. It is also difficult to assess what my students know and what they are capable of when I don't have any. Is my wording effective? Are my questions clear? Can you realistically complete the question? I can quickly see my need to find students and teachers to interact with in order to write these lessons.

All of this and I'm not even half way done writing lesson 1. It's going to be a long semester! But, I am definitely up for the challenge. Especially if it means I'm going to be a better teacher in the end.

- **Remember your objective**
- **Try to challenge and engage your students**
- **Make your questions meaningful**
- **Assess what your students know and what they are capable of**
- **Keep questions in their Zone of Proximal Development**
- **Don't give up!**

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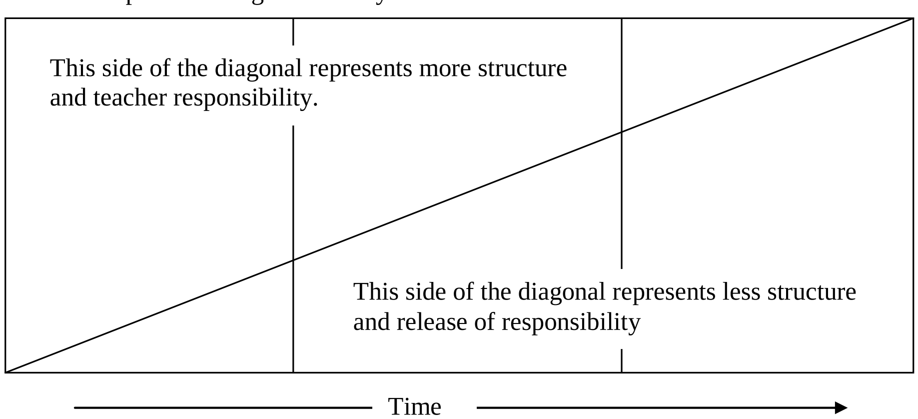
Reflection on Revisions

I've changed the activity now, so I suppose you (whoever *you* are) don't know what the first draft looked like, but that's ok. I met with John today to discuss this first lesson. Going in I knew there were definitely changes I needed to make to it. I was very concerned with:

1. Is this engaging to students?
2. Is this at an appropriate level for algebra I students?
3. Is this too structured? Or maybe not structured enough?

And what did I conclude?

1. Get opinions from students! Do they think it's engaging?
    - I've also changed the activity to make it more engaging in my eyes
    - I previously started the activity with structure and then allowed the students to explore on their own, but now I switched it to exploration first. I have the questions there as an example and guide for students who get stuck, but it's my hope that students will choose to explore two of their own relationships.
2. Who cares?!
    - Ok, it's not that simple. But why should my students be restricted by my low expectations for them?
    - I think that the choice structure allows it to be challenging for advanced students, but also allows structure for those who need it. Hopefully the advanced students will make the choice to explore on their own and the ones who need more help will attempt my questions
3. Release the responsibility
    - In hindsight, I should have expected John to tell me this. After all, I'm pretty sure I copied this diagram into my 329 notes at least 3 times:



    –
    - OK, the diagram may have been slightly more detailed and profound, but you get the point.
    - Since this activity is intended to be for the end of the year, little structure is good!